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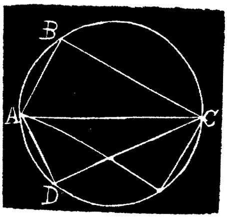
other two sides are also equal; the quadrilateral may be inscribed either with the equal sides adjacent giving $ABCD$, or with the equal sides opposite giving $ABCD$.

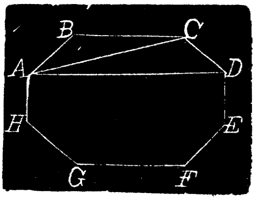


For $AC$ is a diameter of the circumscribing circle in either case.

Proposition vii. To find the polygon of $n$ sides of given perimeter and maximum area.

Let $ABCDEF$ etc. represent the polygon.

Draw the diagonal $AC$, and suppose the sides $AB$ and $BC$ to vary, while the other sides remain fixed in magnitude and position, and we prove, as in Proposition II, that the area of $ABC$, and consequently of the polygon is a maximum, when $AB=BC$. Similarly $BC=CD$, $CD=DE$, etc.



Therefore the polygon is equilateral.

Draw the diagonal $AD$. Then supposing $AB$, $BC$ and $CD$ to vary in *position*, while the other sides remain entirely fixed, we prove, as in Proposition V, that the angles at $B$ and $C$ are equal. We prove in the same way that the angles are equal at $C$ and $D$, at $D$ and $E$, etc; and hence the angles are all equal.

The polygon is therefore both equilateral and equiangular (that is regular) and is inscriptable in a circle.

Proposition viii. Of all Polygons having the same area and the same number of sides, the regular polygon has the maximum perimeter.

For let $P$ be a regular polygon and $M$ any irregular polygon, having the same number of sides and area.

Now if the perimeter of $P$ was equal to that of $M$ then (Prop. VII) the area of $P$ would be greater than that of $M$, but since it is equal to that of $M$ its perimeter must be less.

[To be Continued.]

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# NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M. (Princeton), Ph. D., (Johns Hopkins), Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from the November Number.]

Proposition xii. *Again I say the straight AD somewhere on that side will meet the straight PL (and indeed at a finite, or terminated distance) also in the hypothesis of obtuse angle.*

**Proof.** For, as in the preceding proposition, $DF$ being assumed equal to $AD$, and the just noted perpendiculars let fall, I must show the join $DM$ will be greater than $DF$, or $DA$, and therefore (P.X.) the straight $BM$ will be greater than $AB$.

And in the first place $DM$ will not be equal to $DF$. Otherwise the angle $DMF$ would be equal (Eu. I. 5.) to the angle $DFM$, and therefore greater (P.VIII, in the hypothesis of obtuse angle) than the external angle $XAH$, or its verticle $CAF$.

Wherefore (since the angles $CAM$, $FMA$ are taken equal, as being right) the remaining angle $DMA$ would be less than the remaining angle $DAM$. This is absurd (against Eu.1. 5.), if indeed $DM$ be equal to $DF$, or $DA$.

But neither is $DM$ less than $DF$, or $DA$. Otherwise (Eu.I.18) the angle $DMF$ would be greater than the angle $DFM$, and therefore still greater (in the hypothesis of obtuse angle) than the external angle $XAH$, or its vertical $CAD$, Wherefore again, as above, the remaining angle $DMA$ would be still less than the remaining angle $DAM$. But this is absurd (against Eu.I.18) if indeed $DM$ be lesss than $DF$, or $DA$.

It remains therefore, that the join $DM$ is greater than $DF$, or $DA$, and therefore (P.X.) $BM$ is greater than $AB$. Quod erat hoc loco intentum.

Since therefore, assuming in $AD$ produced the interval $AF$ double the interval $BD$, the perpendicular $FM$ let fall on the transversal $AP$ cuts off from it more than double what is cut off by the perpendicular let fall from the point $D$; more quickly by far in this hypothesis of obtuse angle, than in the preceding hypothesis of right angle, we attain to an interval so great, that from it the perpendicular let fall cuts off a base greater than the designated $AP$ however great.

But this, as in the preceding proposition, could not happen, unless after the meeting of the produced $AD$ with $PL$ in some point; and indeed at a finite, or terminated distance. Quod erat etc.

**Proposition xiii.** *If the straight* $XA$ (*of designated length however great*) *meeting two straights* $AD$, $XL$ *makes with them toward the same parts* (fig.11) *internal angles* $XAD$, $AXL$ *less than two right angles: I say, these two* (*even if neither of those angles be a right angle*) *at length will mutually meet in some point on the side toward those angles, and indeed at a finite, or terminated distance, if either hypothesis holds, of right angle or of obtuse angle.*

**Proof.** For one of the said angles, as $AXL$ suppose, will be acute.

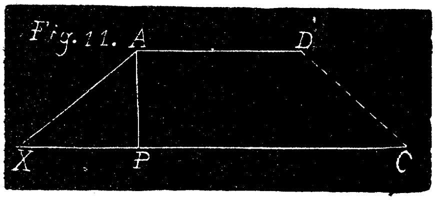
Fig. 11.

Accordingly from the vertex of the other angle is dropped the perpendicular $AP$ on $XL$, which certainly (because of Eu. I. 17.) falls on the side of the acute angle $AXL$. Since therefore in the triangle $APX$, right-angled at $P$, the two acute angles

$PLX$, $PXA$, together are not less (P.IX.) than a right angle, in either hypothesis, of right angle, or of obtuse angle; if these two angles are taken away from the sum of the given angles the then remaining angle $PAD$ will be less than a right angle. Consequently we will be in the case of the two preceding propositions, since it is obvious that one or the other hypothesis holds, either of right angle, or of obtuse angle.

Wherefore the straights $AD$, and $PL$, or $XL$, meet in some point at a finite, or terminated distance on the side noted, as well under the one, as under the other mentioned hypothesis. Quod erat demonstrandum.

[To be Continued ]

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# THE INSCRIPTION OF REGULAR POLYGONS.

**By LEONARD E. DICKSON, M. A., Fellow in Pure Mathematics, University of Chicago.**

## CHAPTER IV.

[Continued from the November Number.]

We will now consider regular polygons the number of whose sides is a *composite* number. The present chapter will be devoted to the case when $n$ is divisible by 3.

*In the regular polygon of* 21 *sides*, the chord $A_7=1$; and $A_3-A_6+A_9=1$, being the chords of a regular 7-gon.

But $A_1-A_2+A_3-A_4+A_5--A_6+A_7-A_8+A_9-A_{10}=1$.

Hence, $(A_1-A_8)+(-A_2+A_5)+(-A_4-A_{10})=-1$.

Now $(A_1-A_8)(-A_4-A_{10})=-[(-A_4-A_{10})+(-A_2+A_5)+2(A_3+A_9)]$.

$(A_1-A_8)(-A_2+A_5)=-[(-A_4-A_{10})+(A_1-A_8)+2(A_3-A_6)]$.

$(-A_2+A_5)(-A_2-A_{10})=-[(A_1-A_8)+(-A_2+A_5)+2(A_9-A_6)]$.

The sum of these 3 products$=-2[(A_1-A_8)+(-A_2+A_5)+(-A_4-A_{10})$
$+2(A_3-A_6+A_9)]=-2$.

Again, $(A_1-A_8)(-A_2+A_5)(-A_4-A_{10})=(A_2-A_5)[(-A_4-A_{10})$
$+(-A_2+A_5)+2(A_3+A_9)]=(A_1-A_8)+(-A_2+A_5)+(-A_4-A_{10})$
$+2(A_3-A_6+A_9)=1$; for $(A_2-A_5)(A_3+A_9)=(A_1-A_8)+(-A_2+A_5)$
$+(-A_4-A_{10})+2=1$; and $(A_2-A_5)^2=2+A_4+A_{10}-2A_3$.

$\therefore$ $(A_1-A_8)$, $(-A_2+A_5)$ and $(-A_4-A_{10})$ are the roots of the cubic

$$x^3+x^2-2x-1=0.$$

But it was shown in Chapter I that the chords $A_3, -A_6, A_9$ of the regular 7-gon are the roots of $x^3-x^2-2x+1=0$.